UPPSALA UNIVERSITET

Institutionen för informationsteknologi
Beräkningsvetenskap

Besöksadress:
MIC hus 2, Polacksbacken
Lägerhyddsvgen 2

Postadress:
Box 337
751 05 Uppsala

Telefon:
018–471 0000 (växel)

Telefax:
018–52 30 49

Hemsida:
http://www.it.uu.se

Department of Information Technology
Scientific Computing

Visiting address:
MIC bldg 2, Polacksbacken
Lägerhyddsvgen 2

Postal address:
Box 337
SE-751 05 Uppsala
SWEDEN

Telephone:
+46 18–471 0000 (switch)

Telefax:
+46 18–52 30 49

Web page:
http://www.it.uu.se

Scientific Computing III

Workout — Linear System of Equations

Mandatory exercises

1. You are going to solve the stationary heat equation for a steel bar whose endpoints are kept at different fixed temperatures. Your Finite Difference discretization leads to the following linear system of equations

$$\underbrace{\begin{pmatrix} -2 & 1 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & & 1 & -2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix}}_{u} = \underbrace{\begin{pmatrix} \alpha \\ 0 \\ \vdots \\ 0 \\ \beta \end{pmatrix}}_{b}$$

   Now, for the case with $N = 4$, LU-factorize the matrix A. Then, with help of your factorization solve the linear system of equations and compute the solution u. Use that $\alpha = -1$ and $\beta = 1$.

2. In this exercise you will apply Jacobi's method to the problem above. For the case of $N = 4$ and $u^{(0)} = [0 \quad 0 \quad 0 \quad 0]^T$: compute $u^{(1)}$ and $u^{(2)}$ by hand, to become familiar with the steps in Jacobi's method.

3. One can argue that neither LU-factorization nor an iterative method like Jacobi's method is suitable for solving the specific problem above, why? When is it suitable to use:

   (a) LU-factorization

   (b) An iterative method, e.g., Jacobi or Conjugate Gradient

4. Stationary iterative methods can be formally expressed as $u^{(k+1)} = Gu^{(k)} + c$. Does the iterative method converge for all $u^{(0)}$ if the matrix G has eigenvalues

   (a) $\lambda = \{-0.35,\ 0.9,\ -0.1,\ 0,\ 0.5\}$?

   (b) $\lambda = \{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1\}$?

Non-mandatory exercises

5. Repeat Exercise 2 for Gauss-Seidel's method.

6. Compute the spectral radius of the iteration matrix for both Jacobi and Gauss-Seidel when applied to the matrix A in Exercise 1 with the case $N = 3$. Which method will converge faster?

7. A matrix A has 5 non-zero diagonals (including the main diagonal). Due to fill-in, the L and U factors of A each have 201 non-zero diagonals. The matrix A is $N \times N$, where $N = 200\,000$. Roughly how many floating point numbers need to be stored when using

   (a) Jacobi's method?

   (b) LU factorization?

   You may assume that each diagonal has the same length and you can neglect everything except the matrices involved.

8. Assume that the matrix A in Exercise 7 has a bandwidth 100. How many floating operations are required with gaussian elimination to solve the system? If we instead use Jacobis method, how many iterations can we do before gaussian elimination becomes more advantageous? Use the case $N = 200\,000$.