

# FACTOR ANALYSIS ADVANCED TOPICS

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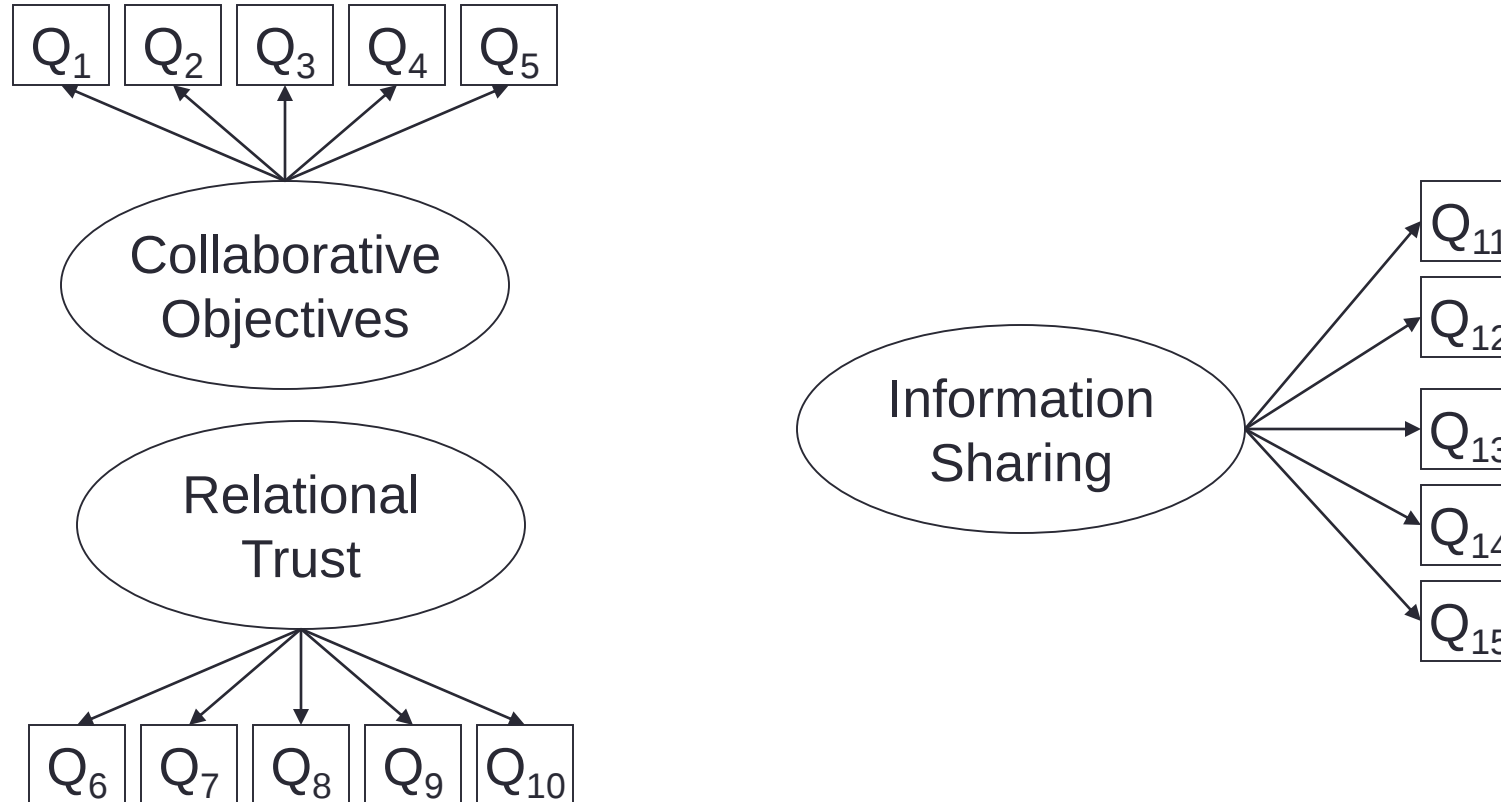
## Chapter 3

# Topics

1. 3-factor example.
2. Adjusting eigen values.
3. Chain of evidence.
4. Reliability analysis.

# 1. A Three-Factor Example

Now we use all 15 indicators.



# Assessment

- Sample size – 315 – good.
- Correlations – good.
- KMO - .938 – good.
- Anti-image – all over .89 – good.
- Communalities – lowest .427 – hmm?
- Cumulative explained variance – 64% - OK.
- Two factors – hmm?

# Communalities

Communalities

|    | Initial | Extraction |
|----|---------|------------|
| CO | .708    | .730       |
| CO | .738    | .752       |
| CO | .743    | .772       |
| CO | .764    | .765       |
| CO | .694    | .720       |
| RT | .602    | .603       |
| RT | .692    | .718       |
| RT | .707    | .717       |
| RT | .736    | .762       |
| RT | .683    | .698       |
| IS | .562    | .525       |
| IS | .531    | .461       |
| IS | .593    | .434       |
| IS | .569    | .456       |
| IS | .481    | .427       |

Low

Extraction Method: Maximum Likelihood.

## 2. Eigen values & Explained Variance

Total Variance Explained

| Factor | Initial Eigenvalues |               |              | Extraction Sums of Squared Loadings |               |              | Rotation Sums of Squared Loadings |               |              |
|--------|---------------------|---------------|--------------|-------------------------------------|---------------|--------------|-----------------------------------|---------------|--------------|
|        | Total               | % of Variance | Cumulative % | Total                               | % of Variance | Cumulative % | Total                             | % of Variance | Cumulative % |
| 1      | 8.456               | 56.376        | 56.376       | 8.036                               | 53.574        | 53.574       | 5.499                             | 36.662        | 36.662       |
| 2      | 1.761               | 11.741        | 68.117       | 1.505                               | 10.036        | 63.610       | 4.042                             | 26.948        | 63.610       |
| 3      | .933                | 6.219         | 74.336       |                                     |               |              |                                   |               |              |
| 4      | .609                | 4.000         | 78.396       |                                     |               |              |                                   |               |              |
| 5      | .473                | 3.152         | 81.548       |                                     |               |              |                                   |               |              |
| 6      | .403                | 2.684         | 84.232       |                                     |               |              |                                   |               |              |
| 7      | .397                | 2.650         | 86.882       |                                     |               |              |                                   |               |              |
| 8      | .364                | 2.427         | 89.309       |                                     |               |              |                                   |               |              |
| 9      | .312                | 2.081         | 91.389       |                                     |               |              |                                   |               |              |
| 10     | .280                | 1.869         | 93.259       |                                     |               |              |                                   |               |              |
| 11     | .257                | 1.710         | 94.969       |                                     |               |              |                                   |               |              |
| 12     | .225                | 1.498         | 96.467       |                                     |               |              |                                   |               |              |
| 13     | .211                | 1.408         | 97.874       |                                     |               |              |                                   |               |              |
| 14     | .179                | 1.192         | 99.066       |                                     |               |              |                                   |               |              |
| 15     | .140                | .934          | 100.000      |                                     |               |              |                                   |               |              |

Almost a 3 factor solution (what we want). The explained variance will also rise.

Extraction Method: Maximum Likelihood.

# Rotated Solution: Eigen at .9

Rotated Factor Matrix<sup>a</sup>

|    | Factor |      |      |
|----|--------|------|------|
|    | 1      | 2    | 3    |
| CO | .290   | .753 | .273 |
| CO | .244   | .793 | .249 |
| CO | .196   | .859 | .119 |
| CO | .335   | .786 | .207 |
| CO | .220   | .773 | .266 |
| RT | .667   | .264 | .296 |
| RT | .751   | .253 | .310 |
| RT | .789   | .253 | .246 |
| RT | .789   | .271 | .291 |
| RT | .757   | .273 | .260 |
| IS | .531   | .254 | .442 |
| IS | .424   | .288 | .502 |
| IS | .272   | .267 | .768 |
| IS | .357   | .254 | .661 |
| IS | .428   | .188 | .531 |

This variable loads higher, and significantly, on the wrong factor.

Remedy

Drop the variable.

Cross-loading

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization

a. Rotation converged in 5 iterations.

# Rotated Solution: Eigen at .9; 4IS

Rotated Factor Matrix<sup>a</sup>

|    | Factor |      |      |
|----|--------|------|------|
|    | 1      | 2    | 3    |
| CO | .754   | .289 | .276 |
| CO | .793   | .247 | .248 |
| CO | .859   | .199 | .117 |
| CO | .786   | .332 | .205 |
| CO | .773   | .218 | .271 |
| RT | .264   | .664 | .306 |
| RT | .253   | .754 | .314 |
| RT | .255   | .786 | .249 |
| RT | .270   | .794 | .298 |
| RT | .278   | .751 | .255 |
| IS | .293   | .422 | .488 |
| IS | .271   | .273 | .753 |
| IS | .253   | .351 | .680 |
| IS | .188   | .421 | .539 |

Now this variable loads high on a different factor from the previous run.

Remedy

Drop the variable.

Cross-loading

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization

a. Rotation converged in 5 iterations.

# Rotated Solution: Eigen at .9; 3IS

Rotated Factor Matrix<sup>a</sup>

|    | Factor |      |           |
|----|--------|------|-----------|
|    | 1      | 2    | 3         |
| CO | .758   | .281 | .275      |
| CO | .797   | .251 | .235      |
| CO | .862   | .207 | 9.145E-02 |
| CO | .788   | .335 | .194      |
| CO | .777   | .204 | .286      |
| RT | .253   | .663 | .312      |
| RT | .258   | .762 | .290      |
| RT | .246   | .787 | .250      |
| RT | .262   | .802 | .274      |
| RT | .286   | .746 | .242      |
| IS | .278   | .321 | .654      |
| IS | .262   | .334 | .721      |
| IS | .197   | .412 | .556      |

This variable is still  
questionable.

Remedy

Drop the variable?

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization

a. Rotation converged in 5 iterations.

### 3. Rotated Solution: Eigen at .9; 2IS

Rotated Factor Matrix<sup>a</sup>

|    | Factor |      |
|----|--------|------|
|    | 1      | 2    |
| CO | .346   | .784 |
| CO | .307   | .815 |
| CO | .220   | .846 |
| CO | .373   | .787 |
| CO | .277   | .806 |
| RT | .726   | .270 |
| RT | .815   | .265 |
| RT | .820   | .249 |
| RT | .846   | .263 |
| RT | .781   | .286 |
| IS | .507   | .362 |
| IS | .539   | .357 |

Two factors?

We need to consider evidence for and evidence against whatever choices we make.

Extraction Method: Maximum Likelihood.

Rotation Method: Varimax with Kaiser Normalization

a. Rotation converged in 3 iterations.

## 4. Reliability Analysis

- Coefficient Alpha (Cronbach's Alpha) is one of the most accepted methods to test reliability.
- In general, the cutoff to accept that a measure is reliable is .7 (Nunnally 1978).

| complex1 | complex2 | complex3 | complex4 | complex5 | var | var | var | var | var | v |
|----------|----------|----------|----------|----------|-----|-----|-----|-----|-----|---|
| 3        | 7        | 7        | 3        | 2        |     |     |     |     |     |   |
| 5        | 2        | 2        |          |          |     |     |     |     |     |   |
| 2        | 5        | 7        |          |          |     |     |     |     |     |   |
| 1        | 1        | 1        |          |          |     |     |     |     |     |   |
| 2        | 1        | 4        |          |          |     |     |     |     |     |   |
| 4        | 2        | 3        |          |          |     |     |     |     |     |   |
| 99       | 7        | 7        |          |          |     |     |     |     |     |   |
| 6        | 6        | 5        |          |          |     |     |     |     |     |   |
| 7        | 7        | 5        |          |          |     |     |     |     |     |   |
| 3        | 1        | 2        |          |          |     |     |     |     |     |   |
| 7        | 5        | 4        |          |          |     |     |     |     |     |   |
| 5        | 5        | 4        |          |          |     |     |     |     |     |   |
| 3        | 5        | 4        |          |          |     |     |     |     |     |   |
| 6        | 6        | 7        |          |          |     |     |     |     |     |   |
| 2        | 1        | 3        |          |          |     |     |     |     |     |   |
| 2        | 1        | 6        |          |          |     |     |     |     |     |   |
| 1        | 1        | 7        |          |          |     |     |     |     |     |   |

Reliability Analysis

Items:

complex1  
complex2  
complex3  
complex4  
complex5

Statistics...

Model: Alpha

Scale label:

OK Paste Reset Cancel Help

var var var var var var var

Statistics...

Help

Reliability Analysis: Statistics

Descriptives for

- ☐ Item
- ☐ Scale
- ☒ Scale if item deleted

Inter-Item

- ☐ Correlations
- ☐ Covariances

Summaries

- ☐ Means
- ☐ Variances
- ☐ Covariances
- ☐ Correlations

ANOVA Table

- ☒ None
- ☐ F test
- ☐ Friedman chi-square
- ☐ Cochran chi-square

☐ Hotelling's T-square

☐ Intraclass correlation coefficient

☐ Tukey's test of additivity

Model: Two-Way Mixed Type: Consistency

Confidence interval: 95 % Test value: 0

Continue Cancel Help

# Alpha if Item Deleted

**Reliability Statistics**

| Cronbach's Alpha | N of Items |
|------------------|------------|
| .660             | 5          |

**Item-Total Statistics**

|          | Scale Mean if Item Deleted | Scale Variance if Item Deleted | Corrected Item-Total Correlation | Cronbach's Alpha if Item Deleted |
|----------|----------------------------|--------------------------------|----------------------------------|----------------------------------|
| complex1 | 15.54                      | 22.275                         | .356                             | .637                             |
| complex2 | 16.04                      | 21.982                         | .423                             | .604                             |
| complex3 | 15.52                      | 25.067                         | .215                             | .694                             |
| complex4 | 16.22                      | 19.890                         | .584                             | .526                             |
| complex5 | 16.21                      | 20.875                         | .520                             | .559                             |

Possible improvement

Still too low.

**Reliability Statistics**

| Cronbach's Alpha | N of Items |
|------------------|------------|
| .694             | 4          |

**Item-Total Statistics**

|           | Scale Mean if Item Deleted | Scale Variance if Item Deleted | Corrected Item-Total Correlation | Cronbach's Alpha if Item Deleted |
|-----------|----------------------------|--------------------------------|----------------------------------|----------------------------------|
| com plex1 | 11.18                      | 16.630                         | .345                             | .716                             |
| com plex2 | 11.68                      | 16.445                         | .409                             | .672                             |
| com plex4 | 11.86                      | 14.304                         | .607                             | .546                             |
| com plex5 | 11.85                      | 14.767                         | .576                             | .568                             |

Possible improvement

# Final Solution

**Reliability Statistics**

| Cronbach's Alpha | N of Items |
|------------------|------------|
| .726             | 3          |

**Item-Total Statistics**

|          | Scale Mean if Item Deleted | Scale Variance if Item Deleted | Corrected Item-Total Correlation | Cronbach's Alpha if Item Deleted |
|----------|----------------------------|--------------------------------|----------------------------------|----------------------------------|
| complex2 | 7.38                       | 9.840                          | .398                             | .809                             |
| complex4 | 7.56                       | 7.986                          | .634                             | .529                             |
| complex5 | 7.55                       | 8.153                          | .628                             | .540                             |

Possible improvement

Multidimensionality versus reliability. How good is good? What is good?